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Stabilization of Adiabatic Quantum Evolution in a Two-Spin XY Model Using the Fast-forward Method

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Abstract

Quantum adiabatic evolution enables a system to follow its target instantaneous eigenstate under slowly varying Hamiltonian parameters, but such a requirement generally demands an excessively long evolution duration. This study aims to accelerate the evolution of an adiabatic system using the fast-forward method, without disturbing the system's initial state trajectory. The system is built from the Hamiltonian model XY and time-dependent parameters, which then determine the regularization term used to maintain the adiabatic trajectory state during the acceleration process. This research methodology is a theoretical study involving mathematical analysis and numerical simulations related to fast-forward and adiabatic dynamics. The research analysis was carried out using a theoretical approach and solving the Time Dependent Schrödinger Equation (TDSE) analytical and numerical calculations assisted by the python program. Simulation results show that the accelerated wave function evolution follows the initial state even though the evolution time is shortened and the velocity parameter is enlarged. In addition, the addition of a fast-forward Hamiltonian has been shown to be able to maintain the stability of the quantum state and suppress non-adiabatic transitions

Keywords: *Adiabatic dynamics, fast-forward, two spins, XY model*

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1. INTRODUCTION

Adiabatic dynamics in quantum mechanics refers to the evolution of a system governed by a Hamiltonian with parameters that change slowly, such that the system remains in its initial state. This principle is described in the quantum adiabatic theorem and serves as the foundation for understanding adiabatic quantum evolution in closed systems (Dodin & Brumer, 2021). Mathematically, the adiabatic condition requires that the rate of change of the parameter be much smaller than the square of the energy gap between levels, so that the probability of a non-adiabatic transition can be neglected (Mori et al., 2024). The requirement for extremely slow parameter changes results in a long evolution time, making it difficult to implement in practice. If adiabatic evolution is accelerated, it will result in non-adiabatic transitions, Berry's (2009) research demonstrates the presence of crossings in accelerated adiabatic trajectories. Long evolution times increase the system's sensitivity to environmental disturbances and fluctuations in external parameters, which can ultimately disrupt the stability of the dynamics (Mohseni et al., 2021). Therefore, the need to shorten the evolution time without violating the adiabatic principle is a key issue in modern quantum dynamics (Guéry-Odelin et al., 2019). In response to these issues, a method for accelerating adiabatic dynamics was developed that enables the acceleration of adiabatic evolution by introducing an additional regularization term into the initial Hamiltonian. This demonstrates that adiabatic dynamics can be accelerated without altering the final state (Masuda & Nakamura, 2010).

Quantum spin systems constitute one of the most fundamental models in quantum mechanics for describing the intrinsic angular momentum of particles and their associated quantum interactions within a two-dimensional Hilbert space through the Pauli matrix formalism (Sakurai & Napolitano, 2017). Interactions between coupled spins give rise to non-classical quantum correlations and parameter-dependent energy spectra, making spin-based Hamiltonians highly relevant for investigating controlled quantum evolution and collective dynamical behavior (Heyl, 2018). In addition, spin models—particularly the XY model—have been widely employed in various areas of modern physics, including studies of spin interactions in semiconductor-based quantum dot systems and spintronics devices, where controllable spin coupling plays a crucial role in the development of quantum information technologies (Chatterjee et al., 2021). In particular, two-spin systems possess a finite Hilbert-space structure that is analytically tractable, allowing the instantaneous eigenvalues and eigenstates of the Hamiltonian to be determined exactly. This analytical accessibility is especially advantageous in fast-forward adiabatic dynamics, where the construction of regularization terms and driving Hamiltonians explicitly depends on the target adiabatic eigenstate. Therefore, the two-spin XY model provides an effective benchmark for examining the capability of the fast-forward method to accelerate quantum evolution while preserving the desired adiabatic trajectory.

The XY model describes the interaction between spin components in the x and y directions via a coupling term that may be anisotropic. The key feature of the XY model lies in its non-diagonal coupling structure, which gives rise to coherent dynamics between energy levels (Sachdev, 2011). To a certain extent, the XY model can also be reduced to an isotropic form related to the Heisenberg model, making it a conceptual bridge between various quantum spin models (Radhakrishnan et al., 2017). In addition, this model has an exact solution for the one-dimensional system via the Jordan–Wigner transformation, which reveals the connection between the spin system and the free fermion system (Franchini, 2017). In modern research, the XY model is widely used in the study of quantum dynamics, entanglement, and quantum control engineering due to its transparent mathematical structure (Amico et al., 2008). The sensitivity of the energy spectrum to variations in the external magnetic field makes this model relevant for analyzing the stability of adiabatic evolution (Dziarmaga, 2010). The XY model has a richer Hamiltonian structure than the Ising model, so it was chosen as a representative system for testing the fast-forward formulation on an anisotropic spin system.

Previous studies have shown that the fast-forward formulation can be applied to various placement systems, including single-particle potential systems and some simple spin models (Masuda & Nakamura, 2010). However, studies that explicitly formulate the fast-forward Hamiltonian for the two-spin XY model and systematically analyze its matrix structure and energy spectrum are still limited in the existing literature. The anisotropic nature of the XY model results in a different coupling structure than the Ising model, thus requiring a more detailed analysis of the differential eigenstate properties of the system in constructing the regularization term. Research by Setiawan (2023) reviewed the three-spin case using different candidate solutions for the regularization term, namely B_z and W_1,, where the use of an external magnetic field makes the solution complex. Meanwhile, this study uses two spins and candidate solutions J_1 and W_1, where the selection of the candidate regularization term is a simple solution that is easy to review explicitly. In the context of modern coordinate dynamical control, an explicit understanding of the Hamiltonian structure is crucial to ensure that the acceleration of evolution does not change the intrinsic energy characteristics of the system (Takahashi, 2017).

2. METHODS

This study is a theoretical physics investigation that combines analytical formulation and numerical simulation to analyze the acceleration of adiabatic evolution in a two-spin XY model using the fast-forward method with time-dependent parameters. The analysis begins with the construction of the initial Hamiltonian of the system, followed by the derivation of a regularization term based on the instantaneous eigenstate to preserve the adiabatic trajectory during the accelerated evolution. Subsequently, the fast-forward Hamiltonian is constructed as the driving Hamiltonian governing the system dynamics. The Time-Dependent Schrödinger Equation (TDSE) is then solved both analytically and numerically to evaluate the evolution of the wave function, the stability of the quantum trajectory, and the effectiveness of suppressing non-adiabatic transitions. The numerical solution of the TDSE is performed through time integration under both the original Hamiltonian and the fast-forward Hamiltonian in order to compare the resulting quantum state evolution trajectories. The overall research procedure adopted in this study is schematically illustrated in the following figure.

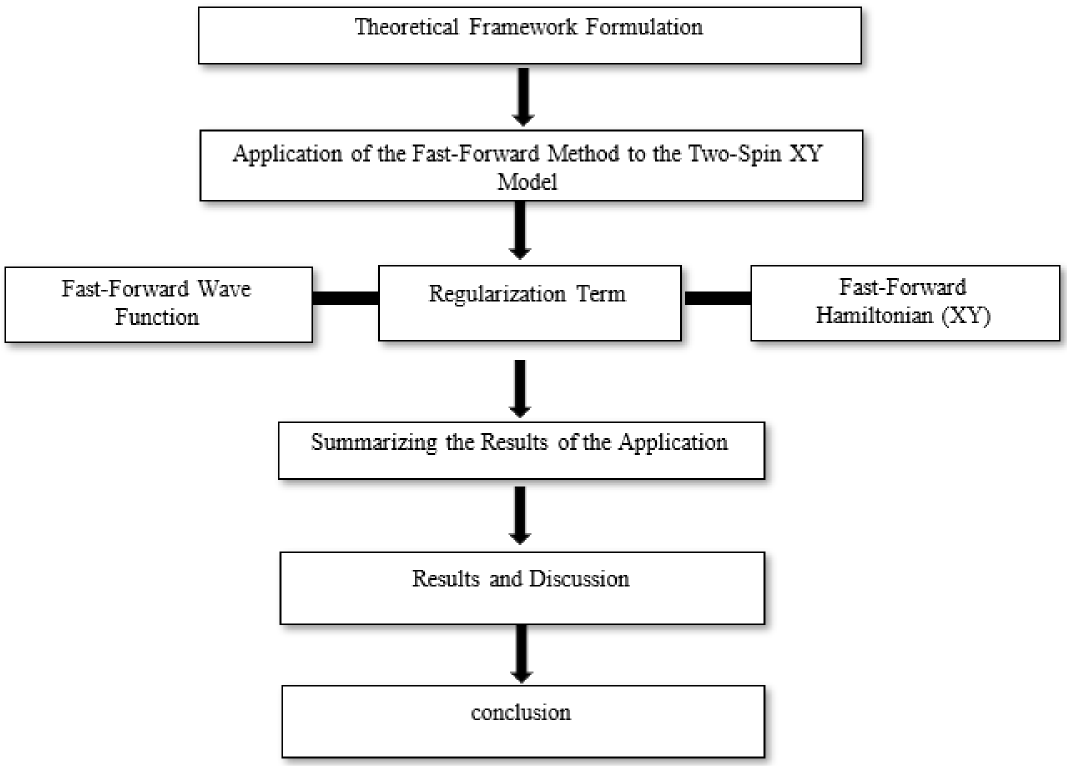


Figure 1. Methodological Framework

The research procedure consists of several systematic stages in accordance with the theoretical framework and analytical approach adopted in this study. The first stage involves the formulation of the theoretical framework, which establishes the fundamental concepts of quantum adiabatic dynamics, the fast-forward method, and spin systems, particularly the two-spin XY model, as the basis for subsequent analysis (Setiawan et al., 2023). The second stage involves the application of the fast-forward method to the two-spin XY model with time-dependent parameters, following the general approach in quantum dynamical control studies (Karuniawan et al., 2024). At this stage, the analytical formulation is carried out through three main components: the derivation of the fast-forward wave function, the determination of the regularization term required to suppress non-adiabatic transitions, and the construction of the fast-forward Hamiltonian for the XY model. These components collectively define the accelerated quantum dynamics of the system and form the basis for further analysis (Gukguk et al., 2024). In the next stage, the formulated Hamiltonians and dynamical equations are analyzed both analytically and numerically by solving the Time-Dependent Schrödinger Equation (TDSE). Numerical simulations are performed to compare the evolution generated by the original adiabatic Hamiltonian and the fast-forward Hamiltonian, with particular emphasis on evaluating the quantum trajectory, system stability, and the suppression of non-adiabatic effects (Nakamura & Masuda, 2022). Finally, the results are systematically analyzed and discussed to assess the effectiveness of the fast-forward method, its consistency with the principles of quantum mechanics, and its limitations, followed by the formulation of the overall conclusions of the study (Apriansyah et al., 2025).

3. RESULT AND DISCUSSION

Result

This section discusses the results of mathematical and numerical analyses of the application of the fast-forward method to a two-spin XY model with adiabatic parameters. The process involves determining the rate of adiabatic dynamics, constructing the fast-forward Hamiltonian based on eigenstate properties, and comparing the matrix structure and energy spectrum with those of the initial Hamiltonian to demonstrate that the acceleration of the dynamics does not alter the intrinsic energy of the system.

In the adiabatic dynamics of a spin system, the first step is to examine the Hamiltonian of the adiabatic system characterized by the adiabatic parameter $R(t)$

$$H_0(R)\begin{pmatrix} C_1(R) \\ \vdots \\ C_N(R) \end{pmatrix} = E(R)\begin{pmatrix} C_1(R) \\ \vdots \\ C_N(R) \end{pmatrix}. \tag{1}$$

This further defines the wave equation in an adiabatic state. Here is the equation,

$$\Psi_0\big(R(t)\big) = \begin{pmatrix} C_1(R) \\ \vdots \\ C_N(R) \end{pmatrix} e^{-\frac{i}{\hbar}\int_0^t E(R(t))dt} e^{i\xi(t)}. \tag{2}$$

Equation 2 yields the Schrödinger equation for an adiabatic system. In this equation, $R(t)$ is the adiabatic parameter, whose value is expressed as a function of time. The form of $R(t) = R_0 + \epsilon t$, where ϵ its value is much smaller than one. Berry Defines the Adiabatic Phase (ξ) as follows,

$$\begin{aligned} \xi(t) &= i\int_0^t dt'\left(C_1^*\frac{\partial C_1}{\partial t} + \cdots + C_N^*\frac{\partial C_N}{\partial t}\right) \\ &= i\epsilon\int_0^t dt'\left(C_1^*\frac{\partial C_1}{\partial R} + \cdots + C_N^*\frac{\partial C_N}{\partial R}\right). \end{aligned} \tag{3}$$

(Berry, 2009)

To have adiabatic properties $\Psi_0(R(t))$ In Equation (2), consistent with the time-dependent Schrödinger equation (TDSE), a modification to the Hamiltonian is required through a regularization process.

$$H_0^{reg}\big(R(t)\big) = H_0 + \epsilon\widetilde{\mathcal{H}}_n\big(R(t)\big) \tag{4}$$

In equation (4) H_0 is the initial Hamiltonian system and $\widetilde{\mathcal{H}}_n$ is the regularization term of the Hamiltonian that depends on the eigenstate under consideration. The TDSE can then be rewritten as

$$i\hbar\frac{\partial}{\partial t}\Psi_0\big(R(t)\big) = \big(H_0 + \epsilon\widetilde{\mathcal{H}}_n\big)\Psi_0\big(R(t)\big). \tag{5}$$

From equation (2), substitute $\Psi_0(R(t))$ by substituting it into the time-dependent Schrödinger equation, we obtain

$$H_0\Psi_0 = E\Psi_0 \tag{6}$$

(Griffiths & Schroeter, 2018).

Then, using equation (6) with an $O(\epsilon^1)$ order

$$\widetilde{\mathcal{H}}_n\begin{pmatrix} C_1(R) \\ \vdots \\ C_N(R) \end{pmatrix} = i\hbar\begin{pmatrix} \frac{\partial C_1(R)}{\partial R} \\ \vdots \\ \frac{\partial C_N(R)}{\partial R} \end{pmatrix} - i\hbar\left(\sum_{j=1}^N C_j^*\frac{\partial C_j}{\partial R}\right)\begin{pmatrix} C_1(R) \\ \vdots \\ C_N(R) \end{pmatrix} \tag{7}$$

(Setiawan et al., 2019).

This equation is the core formulation in the fast-forward scheme, where $\widetilde{\mathcal{H}}_n$ denotes the regularization term dependent on the eigenstate of N. The inclusion of this term allows for the reproduction of the adiabatic wave function, while the time-scaling factor $\Lambda(t)$ is used to accelerate the dynamics without altering the fundamental character of the system. In this study, the cosine function is used as the form of the time scaling factor, with $\Lambda(t)$ defined as

$$\Lambda(t) = \int_0^t \alpha(t')dt', \tag{8}$$

The parameter $\alpha(t)$ is defined as a time-scale factor that satisfies the conditions $\alpha(0) = 1$, $\alpha(t) > 1$ for $0 < t < T_{FF}$, and $\alpha(t) = 1$ for $t \geq T_{FF}$. The wave function in the fast-forward state is defined by

$$\Psi_{FF}(t) = \begin{pmatrix} C_1\left(R\big(\Lambda(t)\big)\right) \\ \vdots \\ C_N\left(R\big(\Lambda(t)\big)\right) \end{pmatrix} e^{-\frac{i}{\hbar}\int_0^t E\left(\left(R\left(\Lambda(t')\right)\right)\right)dt'} e^{i\xi\left(\left(R\left(\Lambda(t)\right)\right)\right)}. \quad (9)$$

In addition, the system dynamics are accelerated by introducing a new time variable so that the target state is reached. $\Psi_0(T)$ can be reached in a shorter time, namely T_{FF}, defined as

$$T = \int_0^{T_{FF}} \alpha(t)dt. \quad (10)$$

The explicit form of the function $\alpha(t)$ on the acceleration interval ($0 \leq t \leq$ TFF) is given by

$$\alpha(t) = \overline{\alpha} - (\overline{\alpha} - 1)\cos\left(\frac{2\pi}{T_{FF}}t\right) \quad (11)$$

(Masuda & Nakamura, 2010),

where $\bar{\alpha}$ represents the average value of $\alpha(t)$, defined as $\overline{\alpha} = T/T_{FF}$. By differentiating with respect to time in Ψ_{FF} as given in Equation (8), and using the relationship

$$\frac{\partial\Psi_0\big(R(\Lambda(t))\big)}{\partial t} = \alpha\epsilon\frac{\partial\Psi_0}{\partial R}, \quad (12)$$

and referring to Equations (1) and (7), the following results are obtained

$$i\hbar\frac{\partial\Psi_{FF}}{\partial t} = \left(v(t)\widetilde{\mathcal{H}}_n\left(R\big(\Lambda(t)\big)\right)\right)\Psi_{FF}$$
$$\equiv H_{FF}\Psi_{FF}. \quad (13)$$

In this context, $v(t)$ is defined as the velocity function derived from $\alpha(t)$ at the asymptotic limit.

$$v(t) = \epsilon\alpha(t)$$
$$= \overline{v}\left(1 - \cos\frac{2\pi}{T_{FF}}t\right), \quad (14)$$

where $\overline{v}$ is the velocity constant, expressed as the product of ε and $\overline{\alpha}$. In the time interval $0 \leq t \leq T_{FF}$, the adiabatic parameter $R\big(\Lambda(t)\big)$ can be rewritten as

$$R\big(\Lambda(t)\big) = R_0 + \epsilon\,\Lambda(t) = R_0 + \int_0^t v(t')dt'$$
$$= \overline{v}\left[t - \frac{T_{FF}}{2\pi}\sin\left(\frac{2\pi t}{T_{FF}}\right)\right]. \quad (15)$$

The H_{FF} Hamiltonian as given in Equation (13) serves as the Hamiltonian controller of the dynamics, while $\widetilde{H}_n$ is the regularization term obtained from Equation (7) and used to accelerate the evolution of the spin system. In this study, a two-spin system is used as a theoretical platform for applying the fast-forward scheme to adiabatic dynamics. The Hamiltonian model is then expressed in the form

$$H_0 = J_1\big(R(t)\big)\sigma_1^x\sigma_2^x + J_2\big(R(t)\big)\sigma_1^y\sigma_2^y + \frac{1}{2}(\sigma_1^z + \sigma_2^z)B_z\big(R(t)\big). \quad (16)$$

The initial Hamiltonian represents an anisotropic XY two-spin system in a magnetic field with an adiabatic parameter $\big(R(t)\big)$ that governs the gradual change in the interaction. The parameters $J_1\big(R(t)\big)$ and $J_2\big(R(t)\big)$ epresent the spin coupling constants in the X and Y directions, respectively, while $B_z\big(R(t)\big)$ represents the external magnetic field along the z-direction, with $J_1 = J_0 - R\big(\Lambda(t)\big)$, $J_2 = R\big(\Lambda(t)\big)$, and $B_z = B_0 - R\big(\Lambda(t)\big)$. The operators σ^x, σ^y, and σ^z are Pauli matrices acting as spin operators on each particle. This form of the Hamiltonian is a standard model widely used in the study of coupled spin systems and quantum adiabatic dynamics because it is capable of capturing the effects of anisotropic interactions and external fields simultaneously (Setiawan et al., 2017).

The operators σ^x, σ^y, and σ^z defined as

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (17)$$

(Griffiths & Schroeter, 2018)

and in this rotation system, the basis $|\uparrow\rangle$ and $|\downarrow\rangle$ are defined

$$\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ \downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (18)$$

From the initial Hamiltonian in the basis $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ obtain

$$H_0\big(R(t)\big) = \begin{pmatrix} B_z\big(R(t)\big) & 0 & 0 & J_1\big(R(t)\big)J_2\big(R(t)\big) \\ 0 & 0 & J_1\big(R(t)\big)+J_2\big(R(t)\big) & 0 \\ 0 & J_1\big(R(t)\big)+J_2\big(R(t)\big) & 0 & 0 \\ J_1\big(R(t)\big)-J_2\big(R(t)\big) & 0 & 0 & -B_z\big(R(t)\big) \end{pmatrix}. \quad (19)$$

The value of $H_0\big(R(t)\big)$ is represented in the form of a four-dimensional matrix, which is obtained from the eigenvalue analysis of the system, namely the eigenvalues of $-J_1 - J_2$, $J_1 + J_2$, $-\sqrt{B_z^2 + J_1^2 + J_2^2 - 2J_1J_2}$ and $\sqrt{B_z^2 + J_1^2 + J_2^2 - 2J_1J_2}$ where the eigenvalues of the ground state are

$$E_0 = -\sqrt{B_z^2 + J_1^2 + J_2^2 - 2J_1J_2}. \quad (20)$$

This review focuses on the ground state of the two-spin XY Hamiltonian, as this state represents the most stable minimum-energy state. By examining the relations $C_2 = C_3 = 0$ and C_1, C_4 are real-valued, and applying the normalization condition ($|C_1|^2 + |C_4|^2 = 1$), further obtain

$$C_1\frac{\partial C_1}{\partial R} + C_4\frac{\partial C_4}{\partial R} = 0, \quad (21)$$

until the adiabatic phase (ξ) = 0. Based on the symmetry properties, the system's regulating term is derived from equation (7)

$$\begin{aligned} i\hbar\frac{\partial C_1}{\partial R} &= \widetilde{\mathcal{H}}_{11}C_1 + \widetilde{\mathcal{H}}_{14}C_4 \\ i\hbar\frac{\partial C_2}{\partial R} &= \widetilde{\mathcal{H}}_{21}C_1 + \widetilde{\mathcal{H}}_{24}C_4 = 0 \\ i\hbar\frac{\partial C_3}{\partial R} &= \widetilde{\mathcal{H}}_{31}C_1 + \widetilde{\mathcal{H}}_{34}C_4 = 0 \\ i\hbar\frac{\partial C_4}{\partial R} &= \widetilde{\mathcal{H}}_{41}C_1 + \widetilde{\mathcal{H}}_{44}C_4. \end{aligned} \quad (22)$$

Next, to complete the process, candidates from the following regularization groups are required (Setiawan et al., 2017)

$$\widetilde{\mathcal{H}} = \begin{pmatrix} \widetilde{J_3} + \widetilde{B_z} & \frac{1}{2}(\widetilde{B_x} - i\widetilde{B_y}) - i\widetilde{W_2} + \widetilde{W_3} & \frac{1}{2}(\widetilde{B_x} - i\widetilde{B_y}) - i\widetilde{W_2} + \widetilde{W_3} & \widetilde{J_1} - \widetilde{J_2} - i2\widetilde{W_1} \\ \frac{1}{2}(B_x + i\widetilde{B_y}) + i\widetilde{W_2} + \widetilde{W_3} & -\widetilde{J_3} & \widetilde{J_1} + \widetilde{J_2} & \frac{1}{2}(\widetilde{B_x} - i\widetilde{B_y}) + i\widetilde{W_2} - \widetilde{W_3} \\ \frac{1}{2}(\widetilde{B_x} + i\widetilde{B_y}) + i\widetilde{W_2} + \widetilde{W_3} & \widetilde{J_1} + \widetilde{J_2} & -\widetilde{J_3} & \frac{1}{2}(\widetilde{B_x} - i\widetilde{B_y}) + i\widetilde{W_2} - \widetilde{W_3} \\ \widetilde{J_1} - \widetilde{J_2} + i2\widetilde{W_1} & \frac{1}{2}(\widetilde{B_x} + i\widetilde{B_y}) - i\widetilde{W_2} - \widetilde{W_3} & \frac{1}{2}(\widetilde{B_x} + i\widetilde{B_y}) - i\widetilde{W_2} - \widetilde{W_3} & \widetilde{J_3} - \widetilde{B_z} \end{pmatrix}. \quad (23)$$

Then select 2 out of 5 variables. (J_1, J_2, J_3, B_z, W_1) obtained from the 2×2 matrix with a nonzero determinant and real solutions. The equation can be written as

$$\begin{aligned} i\hbar\frac{\partial C_1}{\partial R} &= \widetilde{\mathcal{H}}_{11}C_1 + \widetilde{\mathcal{H}}_{14}C_4 \\ i\hbar\frac{\partial C_4}{\partial R} &= \widetilde{\mathcal{H}}_{41}C_1 + \widetilde{\mathcal{H}}_{44}C_4 , \end{aligned} \quad (24)$$

By defining $i\hbar\frac{\partial C_1}{\partial R} = a$ and $i\hbar\frac{\partial C_4}{\partial R} = b$ obtain,

$$\begin{aligned} a &= (\widetilde{J_3} + \widetilde{B_z})C_1 + (\widetilde{J_1} - \widetilde{J_2} - i2\widetilde{W_1})C_4 \\ b &= (\widetilde{J_1} - \widetilde{J_2} + i2\widetilde{W_1})C_1 + (\widetilde{J_3} - \widetilde{B_z})C_4 \end{aligned} \quad (25)$$

The candidate solution for the regularization term is obtained as follows

Table 1. Candidate solutions for regularization coefficients

No	Solutions
1	$J_1 = \dfrac{2i(C_1a + C_4b)}{-4iC_1C_4} = 0$ $W_1 = \dfrac{-C_1a + C_4b}{-4iC_1C_4}$
2	$J_2 = \dfrac{2i(C_1a + C_4b)}{-4iC_1C_4} = 0$ $W_1 = \dfrac{C_1a - C_4b}{-4iC_1C_4}$

No	Solutions
3	$J_3 = \dfrac{2i(C_1a + C_4b)}{2i(C_1^2 + C_4^2)} = 0$ $W_1 = \dfrac{-i(-C_4a + C_1b)}{2(C_1^2 + C_4^2)}$
4	$B_z = \dfrac{-2i(C_1a + C_4b)}{-2i(C_4^2 + C_1^2)} = 0$ $W_1 = -\dfrac{i(C_1a + C_4b)}{2(C_4^2 + C_1^2)}$

In this study, the candidates used were J_1 and W_1. Then, by examining the regularization candidates for that term, the following solution was found

$$\widetilde{\mathcal{H}} = \begin{pmatrix} 0 & 0 & 0 & -i\widetilde{W} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i\widetilde{W} & 0 & 0 & 0 \end{pmatrix}. \tag{26}$$

From equations (4) and (13), we obtain a simplified form of the fast-forward Hamiltonian

$$H_{\mathrm{FF}} = H_0 + v(t)\widetilde{\mathcal{H}} \tag{27}$$

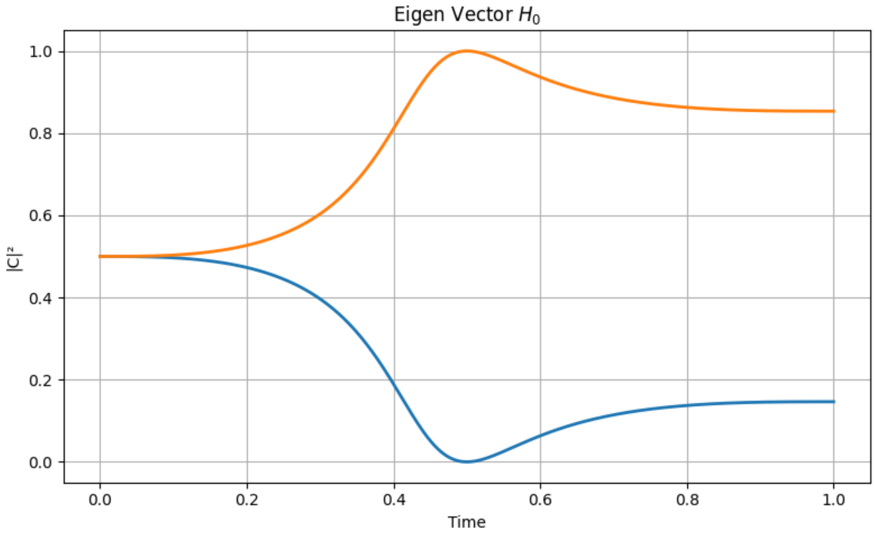
Using equation (27), the fast-forward Hamiltonian can be written as

$$H_{\mathrm{FF}} = \begin{pmatrix} B_z\big(R(t)\big) & 0 & 0 & J_1\big(R(t)\big)J_2\big(R(t)\big) \\ 0 & 0 & J_1\big(R(t)\big) + J_2\big(R(t)\big) & 0 \\ 0 & J_1\big(R(t)\big) + J_2\big(R(t)\big) & 0 & 0 \\ J_1\big(R(t)\big) - J_2\big(R(t)\big) & 0 & 0 & -B_z\big(R(t)\big) \end{pmatrix} + v(t)\begin{pmatrix} 0 & 0 & 0 & -i\widetilde{W} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i\widetilde{W} & 0 & 0 & 0 \end{pmatrix}. \tag{28}$$

Discussion

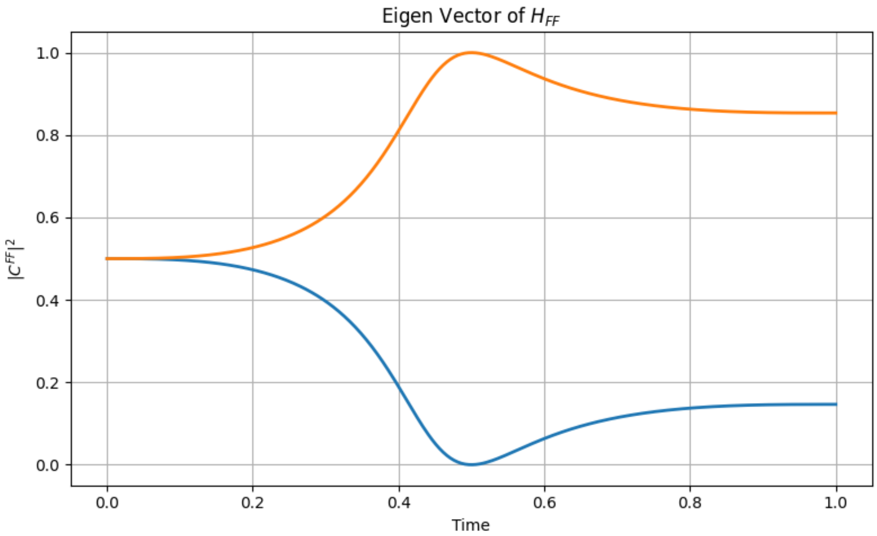
Based on the results of the mathematical derivation, the fast-forward Hamiltonian is obtained as a combination of the initial Hamiltonian of the two-spin XY model and a regularization term, which allows the adiabatic trajectory to remain stable even though the evolution proceeds more rapidly (Guéry-Odelin et al., 2019). In the two-spin XY model system, the trajectory is determined by the eigenstates of the ground state of the initial Hamiltonian $H_0\big(R(t)\big)$, which depends on the adiabatic parameter. Therefore, the analysis in this section focuses on comparing the dynamics of the eigenstates of the initial Hamiltonian H_0 and the fast-forward Hamiltonian H_{FF}, specifically in examining the role of the regularization term in controlling non-adiabatic transitions as discussed in previous studies on quantum spin systems (Setiawan et al., 2017).

The initial Hamiltonian represents an anisotropic XY two-spin model in a magnetic field with an adiabatic parameter $\big(R(t)\big)$ that governs the gradual change in interaction (Sels & Polkovnikov, 2017). The following graph shows the results of solving the Time Dependent Schrödinger Equation, with numerical calculations.



A

Figure 2. Eigenvector plot with numerical solution of the TDSE for the initial Hamiltonian H_0 The line — (blue) corresponds to C_1 while the line — (orange) corresponds to C_4



B

Figure 3. Eigenvector plot using the fast-forward numerical solution of the Hamiltonian TDSE. The line — (blue) corresponds to C_1 while the line — (orange) corresponds to C_4

The first graph shows the evolution of the ground state eigenstate derived from the initial Hamiltonian $H_0(R(t))$ without the addition of a regularization term. The parameters used for the first graph are $J_1 = J_0 - R(\Lambda(t))$, $J_2 = R(\Lambda(t))$, $B_z = B_0 - R(\Lambda(t))$, $\tilde{v} = 10$, $T_{FF} = 1$, $J_0(R_0 = 0) = 10$ and $B_0 = 0$ while the second graph uses $J_1 = J_0 - R(\Lambda(t))$, $J_2 = R(\Lambda(t))$, $B_z = B_0 - R(\Lambda(t))$, $\tilde{v} = 100$, $T_{FF} = 1$, $J_0(R_0 = 0) = 10$ and $B_0 = 0$.

In terms of the energy spectrum structure, both the initial Hamiltonian (H_0) and the fast-forward Hamiltonian (H_{FF}) exhibit identical eigenvalue patterns throughout the adiabatic parameter evolution. This pattern is reflected in the eigenstate plots, which show similar trajectories without any changes in the curve structure or shifts in the final states. This similarity indicates that the inclusion of the regularization term does not modify the system's energy spectrum but preserves the fundamental characteristics of the two-spin XY model Hamiltonian (Kolodrubetz et al., 2017).

Based on the probability-versus-time plot, the evolution of the system with the fast-forward Hamiltonian reaches the target state in a shorter time than the initial Hamiltonian. This acceleration is achieved through a modification of the Hamiltonian in equation (27), where the counterdiabatic term ensures that the system continues to follow the adiabatic path. The agreement of the evolution results with the adiabatic solution is demonstrated through the fidelity ($F(t)$), which is expressed as follows

$$F(t) = \left|\left\langle \Psi_{FF}(t) \mid \Psi_0\left(R\left(\Lambda(t)\right)\right)\right\rangle\right| = \left|\sum_j C^*_{FF,j}(t)\ C_j\left(R\left(\Lambda(t)\right)\right)\right| \quad (29)$$

(Setiawan et al., 2017)

with an accuracy of 1- ϵ where $0 \leq \epsilon \leq 10^{-6}$ over the time interval 0≤ t ≤T_{FF}. This indicates that the acceleration does not alter the final state, thus demonstrating that the fast-forward method in the XY model accurately and stably accelerates the adiabatic evolution.

Compared with the previous research by Setiawan (2023), several important differences can be identified. First, the previous study examined a three-spin system, whereas the present study focuses on a two-spin system. The use of a two-spin system allows for a simpler and more controlled analysis, enabling the instantaneous eigenstates to be determined more explicitly. This provides a clear advantage in the construction of fast-forward dynamics, both analytically and numerically. Second, there is a difference in the selection of regularization terms. The previous study employed B_z and W1, where B_z represents an external magnetic field that tends to have a more complex form in its mathematical formulation. In contrast, the present study adopts J_1 and W_1, where J_1 is directly associated with the intrinsic spin–spin coupling in the XY model. This choice produces a regularization term that is simpler and easier to handle, both in analytical derivations and numerical implementation. Therefore, the construction of the fast-forward Hamiltonian becomes more efficient without reducing the level of accuracy in preserving the adiabatic trajectory. This is evidenced by the high fidelity values and the stable evolution of the system obtained in this study. Accordingly, this work not only confirms the effectiveness of the fast-forward method in accelerating adiabatic evolution, but also demonstrates that the proper selection of regularization terms can improve the ease of implementation and the computational efficiency of the method in quantum spin systems.

4. CONCLUSION

This study successfully demonstrates that the fast-forward method can be effectively applied to a two-spin XY model system to accelerate adiabatic evolution without altering the intrinsic structure of the system's energy spectrum. The inclusion of a regularization term in the driving Hamiltonian enables the system to closely follow the instantaneous eigenstate trajectory, even when the evolution time is significantly reduced. This behavior is supported by the agreement between the numerical trajectories of the original Hamiltonian and the fast-forward Hamiltonian, as well as by fidelity values approaching unity, indicating the stability and accuracy of the resulting quantum evolution.Compared to previous studies, this work provides a more specific contribution through its application to a simpler and more controlled two-spin system, allowing both analytical formulation and numerical implementation to be carried out more explicitly. In addition, the selection of the regularization terms J_1 and W_1 results in a more simplified and efficient construction of the fast-forward Hamiltonian compared to earlier approaches that involve an external magnetic field B_z. These findings indicate that an appropriate choice of regularization terms not only preserves the performance of adiabatic acceleration but also improves the practicality of implementing the method in quantum spin systems.

Thus, this research not only confirms the effectiveness of the fast-forward method in quantum dynamical control but also offers a more operational formulation for simple spin systems. For future work, the analysis can be extended to higher excited energy levels as well as to spin models with a larger number of particles in order to further test the generality of the method.

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